HW Two, MTH 418, Spring 2016

Ayman Badawi

QUESTION 1.   (i) Given A graph $H = C_n$ for some positive integer $n \geq 3$, where $diam(H) = 7$. Find all possible values of n. If H is a bipartite graph, what are all values of n?

(ii) If C_n is not a bipartite graph and $diam(C_n) = m$ for some odd integer m. Find all possible values of n (write all values of n in terms of m)

(iii) Construct a connected graph with minimum number of vertices such that each vertex is of degree 3.

(iv) Fix an integer m. Let H be a connected graph with minimum number of vertices, say n, such that each vertex is of degree m. What is the value of n (in terms of m)? Prove your claim (you may use math induction!)

(v) Let H be a graph of order 5 and of size 3. How many nontrivial spanning subgraphs does H have? (i.e., Note that if $F = H$, then F is a trivial spanning subgraph of H)

(vi) Let H be a graph of order n and of size m. How many nontrivial spanning subgraphs does H have? prove your claim.

(vii) Consider the graph $H = K_{3,4}$ with the associated two sets of vertices A, B such that $|A| = 3$ and $|B| = 4$. Let D be the induced subgraph of H with vertex set = A. Construct the graph joint of E_2 and D, Does the graph look familiar? Is the graph joint of E_2 and H a bipartite graph?

(viii) Consider the graph $H = Q_4$ with vertex set V. Let $v = 0001 \in V$. Let $F_1 = \{w \in V | d(v, w) = 2\}$ and $F_2 = \{w \in V | d(v, w) = 3\}$. Find F_1 and F_2. Construct the induced subgraph, say M, of H with the vertex set $F_1 \cup F_2$? Can you say something about M?

(ix) Give me an example of two graphs, each is of order 6, both have the same associated non-increasing sequence on the degrees of the vertices, but one of them is disconnected while the other is connected.

Due date: Thursday at noon March 3 ,2016 Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com